

# Choosing Training Images and Checking Realizations with Multiple Point Statistics

Jeff Boisvert<sup>1</sup>, Michael J. Pyrcz<sup>2</sup>, and Clayton V. Deutsch<sup>1</sup>

<sup>1</sup>Center for Computational Geostatistics (CCG) – University of Alberta

<sup>2</sup>Chevron Energy Technology Company, Houston, TX

*Selecting a training image (TI) that is representative of the reservoir of interest is essential for an effective application of multiple point statistics. It is often possible to narrow potential TIs to a particular group of images based on the available geological knowledge; however, choosing a specific TI is often difficult and is largely subjective. Significant differences between the spatial statistics of local conditioning and a TI may lead to artifacts in MPS. A methodology is presented that compares the distribution of runs and the multiple point histogram from available exploration data to the same statistics in TIs. The difference in the multiple point statistics can be used to select the TI that is most representative of the data set. This procedure is used to select a TI for a set of 32 deepwater wells provided by Chevron. An extension of this technique is to use the multiple point comparison to rank realizations. To demonstrate this, 30 multiple point realizations are generated using single normal equations simulation (SNESIM) and compared to 30 realizations generated using sequential indicator simulations (SIS). The realizations are then ranked based on their similarity to the parent TI and it is found that the multiple point realizations are more similar to the original TI. It is anticipated that ranking realizations would be used to select appropriate realizations for further analysis, such as flow simulation, uncertainty calculations, or reserve estimation and for checking the reproduction of input statistics for model validation.*

## Introduction

There are a number of implementation issues that have not been addressed with MPS. Focus has been placed on how to create an algorithm that reproduces the multiple point statistics borrowed from a TI. This is extremely important, and is at the core of any application of multiple point statistics. However, there are other pressing issues such as the selection of an appropriate TI. Just as the effectiveness of sequential Gaussian simulation (SGS) depends on a valid variogram model, the effectiveness of multiple point simulation depends on a valid TI. In this context, a valid TI is defined as an exhaustive, gridded set of data. Exhaustive translates to having a sufficiently large TI such that all necessary multiple point statistics are inferable. The necessity for the TI to be gridded is founded in the reliance of multiple point algorithms on gridded TIs; this is a practical limitation rather than a theoretical one. In addition to these constraints, a TI must be representative of the area of interest, inasmuch as it possesses heterogeneities present in the geology and that have a significant impact on the transfer function. Just as the variogram contains the two point relationships in a data set, the TI contains the multiple point statistics that will be reproduced in the realizations. Variograms are often inferred from available exploration data and similar reservoirs. Unfortunately, the number of multiple point statistics required to satisfy the exhaustive constraint is much larger than any normal exploration data set. Other sources are required to infer the necessary multiple point statistics.

A geologically realistic source for an appropriate TI is extensively sampled reservoirs. Many problems hinder the use of exhaustive data for multiple point statistical inference:

- Lack of flexibility – There is very little opportunity to adjust the data to fit the area of interest. Properties such as anisotropy, geological units, complex geometry or proportions of the extensively sampled area must be representative of the area of interest and are difficult to alter to fit new areas. For example, if the production data is available for a fluvial reservoir, but the sinuosity of the channels is slightly greater than the reservoir of interest, it would be difficult to adjust this and use the data as a TI.
- Lack of availability – It is rare that one company will develop two areas that are sufficiently similar to use data from the older area to infer multiple point statistics; this, coupled with companies reluctance to share data in its original form, makes obtaining extensively sampled data difficult.
- Lack of gridded data – Even when production data is available it is seldom on a regular grid. Current multiple point simulation algorithms are limited to using TIs that are defined on a regular grid because of the difficulty in defining a multiple point template for an unstructured grid. Production data must be altered to fit a regular grid and this can pose problems. Alternatively, dense production data could be used to generate a very detailed and accurate geostatistical model on a regular grid.

The difficulties in using production data for TIs has resulted in the use of process based modeling to generate TIs that have realistic geological features. This has been done by Pyrcz and Deutsch (2003), Pyrcz et al. (2005) and Pyrcz and Strebelle (2006) for fluvial and deepwater TIs. Mimicking geological processes generates TIs that are exhaustive and contain the desired geological features. However, these process based techniques generate many TIs with different properties such as different net to gross ratios (NTG), sinuosity, channel size, depositional settings, etc. When presented with a host of potential TIs, the geostatistician must decide which are more appropriate for their reservoir. Often TI selection is done subjectively based on the available geological knowledge. The methodology presented here attempts to remove the subjective aspect of selecting a TI by comparing the multiple point statistics of TIs to the exploration data available. As mentioned previously, there is not sufficient exploration data to infer all multiple point statistics but there is often a sufficient number of data to make an accurate comparison between potential TIs and the data. This can be used to rank TIs on their similarity to the reservoir of interest, thus helping to select a TI that is representative of the area of interest.

An extension of this concept is to use the same methodology after simulation to rank multiple point realizations based on their similarity to the TI selected. This would occur after the selection of an appropriate TI and after multiple point realizations have been simulated. Once realizations are available, they can be compared to the original TI to determine how well the algorithm reproduced the multiple point statistics of the TI.

## **Background**

Multiple point statistics are an emerging solution to the limitations inherent in traditional geostatistical techniques. A description of the variogram is given to highlight these limitations. Generating a TI is only the initial step in estimating or simulating with multiple point techniques. Once a TI has been selected the multiple point statistics from that TI must be reproduced in a set of realizations. These statistics can never be honored exactly, or the resulting simulations would

be identical to the TI; however, the multiple point statistics of the realizations should be as close as possible to the TI while honoring the available data for the reservoir of interest. Two types of multiple point statistics, the distribution of runs and the multiple point histogram, will be described as they will be used to rank TIs and realizations.

#### *Variogram Use and Limitations*

The variogram is commonly used to define the covariance between two locations in space separated by a distance  $\mathbf{h}$ . The two locations are defined as  $\mathbf{u}$  and  $\mathbf{u} + \mathbf{h}$ , by definition the variogram ( $\gamma$ ) is:

$$\begin{aligned} 2\gamma(\mathbf{h}) &= E\{[Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})]^2\} \\ &= E\{[Z(\mathbf{u})]^2\} - 2E\{Z(\mathbf{u}) \cdot Z(\mathbf{u} + \mathbf{h})\} + E\{[Z(\mathbf{u} + \mathbf{h})]^2\} \end{aligned} \quad [1]$$

This can be used to calculate the covariance between two points. The covariance between points separated by a distance  $\mathbf{h}$  is:

$$C(\mathbf{h}) = E\{Z(\mathbf{u}) \cdot Z(\mathbf{u} + \mathbf{h})\} - E\{Z(\mathbf{u})\} \cdot E\{Z(\mathbf{u} + \mathbf{h})\} \quad [2]$$

The variance ( $\sigma^2$ ) is defined as the covariance at a distance  $\mathbf{h}=0$ :

$$C(0) = \sigma^2 = E\{[Z(\mathbf{u})]^2\} - E\{Z(\mathbf{u})\}^2 \quad [3]$$

Combining equations 1,2 and 3 yields:

$$\gamma(h) = C(0) - C(\mathbf{h}) \quad [4]$$

In this way the covariance between points separated by a distance can be calculated from the variogram.  $\gamma(\mathbf{u}+\mathbf{h})$  for multiple lag distances,  $\mathbf{h}$ , are often calculated from the available exploration data and then modeled with a positive definite function. The variogram model is most commonly used to estimate a variable in a domain using Kriging or to generate a series of realizations using sequential Gaussian simulation. Kriging is a linear estimator that assigns weights  $\lambda_i$ , to the  $i^{th}$  data point  $Z(\mathbf{u}_i)$ , to estimate at a given location  $\mathbf{u}$ :

$$Z^*(\mathbf{u}) = \sum_{i=1}^n \lambda_i \cdot Z(\mathbf{u}_i) \quad [5]$$

The weights generated by Kriging minimizes the error variance: the expected squared difference between the estimate at a location,  $Z^*(\mathbf{u})$ , and the true value at that location,  $Z(\mathbf{u})$ . This estimate has less variance than the original variable. This smoothness is well known and is addressed using sequential Gaussian simulation (SGS) to add a random component to the Kriged estimate that corrects for the smoothing effect of Kriging. Kriging, SGS and the smoothing effect are well documented in literature, for example see Journel and Huijbregts (1978).

Variogram based techniques often assume the variable of interest follows a multi Gaussian distribution. Often realistic results can be generated under a multi Gaussian framework using transformations, however, any parametric approach will be limited in that it cannot reproduce complex features unless they can be defined by the parameters, which is seldom the case. The limitations inherent in a multi Gaussian assumption where the variogram is used to define the two point relationship is that the curvilinear features and ordering relationships that are often present in geological formations cannot be reproduced. For many applications, excluding the curvilinear features is critical. For example, removal of nonlinear features may drastically change

connectivity of net facies. The need to reproduce these nonlinear features in geostatistical realizations motivates the use of multiple point statistics.

### *Multiple Point Statistics*

Multiple point statistics are any type of statistic that considers the relationship between more than one data point. For this paper, multiple point statistics are considered to be statistics that consider the relationship between more than two points (i.e. statistics beyond the variogram). Multiple point statistics are often compared to two point statistics and the difference will be made clear.

There are many different multiple point statistics, such as the connectivity function, the multiple point histogram and the distribution of runs. The multiple point histogram and the distribution of runs will be used to select appropriate TIs and rank realizations.

### *The Distribution of Runs*

Runs are a type of multiple point statistic that can be applied to a 1-D string of values, such as a well. The concept is to count the number of consecutive occurrences of a particular category. This is illustrated in Figure 1.

The distribution of runs for this example would be 3 runs of length one, 2 runs of length two, and 1 run of length three. The cumulative distribution of runs can be found by noting that each higher order run contains all lower order runs. For example, the run of three gray blocks contains two runs of length two and three runs of length one; therefore, the cumulative distribution would be 10 runs of length one, 4 runs of length two and 1 run of length three. Figure 2 shows this distribution of runs.

It is not necessary that the spacing of the string of numbers be equidistant and parallel, but this simplifies the analysis. Runs will be calculated on wells and it is realistic to consider straight wells with equally spaced samples.

### *The Multiple Point Histogram*

The traditional histogram counts the frequency of times a particular continuous variable falls in a bin, or counts the frequency of times that a particular indicator variable occurs. The multiple point histogram counts the frequency of times a multiple point configuration occurs. Consider a 4 point configuration that could take two different values, there are a total of 16 ( $2^4$ ) unique configurations.

Considering this four point configuration, or template, the multiple histogram for a gridded image can be calculated. The number of times that each configuration in Figure 3 occurs in the image is counted and the bins of the histogram correspond to the different configurations. Consider the image in Figure 4 with its multiple point histogram shown in Figure 5.

Note that the ordering of the bins in Figure 5 is entirely arbitrary, so long as the bins are ordered consistently there is little confusion. Considering a larger template or more categories significantly increases the number of bins in the multiple point histogram, making visualizing the multiple point histogram for large templates challenging. There is no limitation on the shape or size of the histogram other than the computational effort required to deal with the large number of bins: other possible configurations are shown in Figure 6.

### *Using Multiple Point Statistics*

The multiple point histogram is commonly used in multiple point simulation algorithms. Traditionally, the variogram has been modeled based on exploration data; this is not possible

when using the multiple point histogram because of the volume of data necessary to accurately infer all bins of the multiple point histogram. An alternative source for the inference of multiple point statistics is needed. Selecting a TI to be representative of the area of interest has become the most common method of inferring the necessary multiple point statistics.

It may be possible to obtain the multiple point statistics by fitting a function based on the available exploration data. However, the non-linearity and the dimensionality of the multiple point histogram makes this a difficult task. Consider that a 16 point statistic with 10 categories, results in  $10^{16}$  bins with each bin unrelated to adjacent bins. This would be a difficult function to fit. For these reasons, it is common to select a TI that is representative of the reservoir of interest to infer the necessary multiple point statistics. It is also necessary that the TI is exhaustive to ensure all statistics, such as the bins of the histogram, can be informed.

Techniques available for multiple point simulation include: using the single normal equations (Guardiano and Srivastava, 1993 and Strebelle, 2002); using simulated annealing with the multiple point histogram as the objective function (Deutsch, 1992); updating conditional distributions with multiple point statistics (Ortiz and Deutsch, 2003); and, training neural networks on training images for multiple point simulation. Refer to these sources for details on multiple point simulation techniques.

### **Proposed Methodology**

Fluvial and deepwater TIs (Pyrzcz and Deutsch, 2003) will be compared to a data set provided by Michael Pyrcz (Chevron) to demonstrate the methodology for selecting TIs. It is intended that this methodology be used after the set of TIs has been narrowed based on the available geological knowledge. Lower order statistics, such as the variogram, should also be used when selecting a TI. This is commonly done and is not considered here, only the multiple point statistics are used to demonstrate the proposed methodology.

The proposed methodology is a comparison of the absolute difference between the runs distribution or multiple point histogram of the TIs and the available exploration data. TIs are then ranked based on this difference. Only one dimensional multiple point statistics will be considered because well data contains many samples in the vertical direction. As a result, any multiple point statistic used to compare TIs and exploration data will be better informed in the vertical direction.

Following the selection of an appropriate TI, the concept of ranking realizations using the multiple point histogram or the distribution of runs is explored.

#### *Using Runs to Select TIs*

Once the cumulative distribution of runs is calculated for the available exploration data it can be compared to the distribution of runs for all potential TIs. This is done by calculating the absolute difference between the distributions and summing over all runs lengths, see Figure 7.

The difference between distributions may be calculated as:

$$diff = \sum_{l=1}^n |f_l^* - f_l^{**}| \quad [6]$$

Where *diff* is the difference between the distributions, *l* is the length of runs, *n* is the maximum length of runs,  $f_l^*$  is the cumulative frequency of the well data for a runs length of *l* and  $f_l^{**}$  is the cumulative frequency of the TI for a runs length *l*. The smaller the summed difference is, the

more alike the two runs distributions are. This gives a quantitative measure that can be used to rank potential TIs.

There are a number of practical issues with using the runs distribution to select TIs. First, the scale of the features in the TI may be much larger or smaller than the scale of features in the available data. This can be corrected by downscaling the TI or generating new TIs with the appropriate dimensions if possible. The second issue arises from the distribution of runs being continuous. For example, a runs length of 100 would be very similar to a runs length of 99. If the absolute difference between runs lengths were calculated this would artificially inflate the difference between the distributions. This is dealt with by fitting an exponential function to the local runs distribution when calculating the difference, smoothing out the effect of similar runs lengths. Moreover, using the cumulative distribution of runs also minimizes this problem as a runs length of 100 would contain two 99 runs. A third practical consideration is the definition of the facies in the TI and the exploration data. In general, using fewer facies will increase the number of long runs in the image and reduces the number of shorter runs. This emphasizes the need to have TIs with the identical facies definitions as the well data in order to make a consistent comparison of statistics. A binary net/non-net set of facies is chosen for consistency.

#### *Using the Multiple Point Histogram to Select TIs*

A similar approach can be taken to compare potential TIs to data based on the multiple point histogram. The methodology is identical to using the runs distribution but the difference is taken as the sum of the absolute difference between each multiple point bin.

When using the multiple point histogram to compare distributions there is no exponential fit used, as the multiple point histogram is not continuous and adjacent bins are unrelated. One implementation issue when using the multiple point histogram is dealing with the uninformed bins. Uninformed, or zero bins often pose a problem in multiple point statistics as they can be missing, not necessarily zero. It is often the case that a particular bin is not informed because of the sparse exploration data available, as opposed to the particular configuration the zero bin corresponds to actually occurring with zero frequency in the reservoir. Therefore, two cases can be considered: i) the naïve difference between the multiple point histogram, and ii) only summing the difference between bins when both bins are informed, effectively ignoring the zero bins that occur in the well data.

Case i) naïve difference:

$$diff = \sum_{i=1}^n |f_i^* - f_i^{**}| \quad [7]$$

Case ii) ignore zero bins:

$$diff = \sum_{i=1}^n \begin{cases} \text{if } f_i^* = 0, & 0 \\ \text{if } f_i^{**} = 0, & 0 \\ \text{otherwise} & |f_i^* - f_i^{**}| \end{cases} \quad [8]$$

Where *diff* is the difference between the distributions, *i* is the bin number, *n* is the maximum number of bins,  $f_i^*$  is the frequency corresponding to bin *i* for the wells and  $f_i^{**}$  is the frequency corresponding to bin *i* for the TI. The smaller the summed difference is, the more alike the two

multiple point distributions are. In case ii, the difference at each bin is only summed if  $f_i^*$  and  $f_i^{**}$  are informed. Figure 8 shows two multiple point distributions for a square 4 point template. For the remainder of this paper only case i will be considered. It was found that ignoring the zero bins had little effect on the examples presented in this paper.

For the example shown in Figure 8 the difference between the frequencies for all 16 configurations would be summed to obtain the difference in the multiple point histogram. Note the four zero bins in the TI, for considering case ii, the difference would be calculated ignoring these bins.

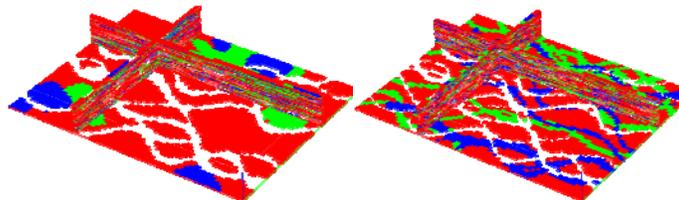
Having identical facies when comparing multiple point statistics is extremely important. The size (number of bins) of the multiple point histogram is dependent on the number of facies present, therefore, facies must be consistently defined across all potential TIs and exploration data.

When using the multiple point histogram to compare well data to a set of potential TIs it is important to understand the nature of the 1-D exploration data. A multiple point configuration, as shown in Figure 8, would not be possible because only 1-D data is available in the form of exploration wells. Therefore, a one dimensional multiple point template must be used to compare well data to TIs, see Figure 9. If comparing geostatistical realizations to their parent TI this is not an issue because of the 3-D nature of the realizations.

### Case Study: Chevron Well Data Comparison

Chevron provided a data set consisting of 32 wells from a West African reservoir. The depositional environment of the reservoir is deepwater, which will help to narrow the selection of a TI. TIs from the fluvial/deepwater library (Pyrcz and Deutsch, 2003) will be compared to the 32 wells using the methodology presented above. The three TIs Chevron selected as being representative of the area will be included in the comparison. It is important that the facies for all TIs and the data are consistent; however, the Chevron data, fluvial/deepwater library and the three TIs from Chevron contain different facies definitions. The TIs and data were converted to net and non-net categories for consistency. For the fluvial TIs in the training image library, the flood plain and abandoned channel fill categories were considered to be non net and the remaining categories net.

294 TIs from the fluvial/deepwater TI library (Pyrcz and Deutsch, 2003) and the three TIs from Chevron were compared to the Chevron data set. Each Chevron TI is 78x59x116 and modeled after a deepwater setting characterized by channels and lobes, see Figure 10. The distribution of runs for the TIs provided by Chevron are shown in Figure 11. Each TI in the fluvial/deepwater TI library is 256x256x128.



All 297 TIs are ranked according to the differences in the runs distribution and the multiple point histogram. Figure 12 shows the ranking of each TI. It is known that the 32 wells originated from

a deepwater setting, thus, it is expected that the deepwater TIs would be ranked higher. This is the case for ranking with the runs distribution and ranking with the multiple point histogram.

When the multiple point histogram is used to rank the TIs, ten of the top ten ranked TIs are from a deepwater setting, with two being the Chevron TIs. When the runs distribution is used to rank the TIs the top 29 are from a deepwater setting with three being the Chevron TIs. This is promising as the methodology has selected the correct depositional setting. It is also encouraging that the TIs that were selected by Chevron rank highly as these were deemed representative of the well data.

Figure 13 shows that, in general, the ranking statistics agree well with each other. The high correlation, 0.9, seen between the two ranking methods is encouraging and would reinforce the selection of a particular TI.

Comparing multiple point statistics is a useful tool in helping to select TIs but a visual comparison of the TIs to the well data would further reinforce the selection of a TI. It is difficult to compare a large model, such as a TI, to well data; therefore, 32 pseudo wells were drilled into each TI and are displayed in Figure 14 so that a visual comparison to the 32 wells can be made. Shown are the highest and lowest ranked TIs using the ranking methods presented above, as well as the three TIs proved by Chevron.

This visual inspection reveals that the 1<sup>st</sup> ranked TI based on the runs distribution has a high net to gross ratio (NTG). The NTG for the TIs shown in Figure 14 are given in Table 1. Both top ranked TIs have NTG ratios that are slightly off of the well data.

**Table 1: NTG for select TIs.**

<b>TI</b>	<b>NTG</b>
Chevron Well Data	0.49
Ranked 1 <sup>st</sup> by runs	0.69
Ranked 1 <sup>st</sup> by MPS	0.36
Ranked 297 <sup>th</sup> by both	0.012
Chevron TI #1	0.48
Chevron TI #2	0.48
Chevron TI #3	0.48

**Discussion: Using MPS to select TIs**

Selecting TIs based on a multiple point statistical comparison is meant to narrow the list of possible TIs. It is important to understand the place of the runs distribution and the multiple point histogram in selecting TIs. Based on prior experience it is believed that runs are more stable for sparse data while the multiple point histogram does well when there is sufficient data to inform the bins. Each well in the Chevron data contains an average of 118 logs for a total of 3773 data points. This should be a sufficient number of data to generate a multiple point histogram that would outperform the runs distribution for TI selection. If there are too few wells, and an informed multiple point histogram is not attainable, runs would likely be more robust. Both statistics should be used to help select TIs. The final selection should be made on the bases of all available information including the NTG ratio, histogram, variogram, geological knowledge and experience with similar formations. It has been common practice to use these sources of information to select TIs, however, the methods to compare TIs and exploration data are usually

subjective. We have shown that multiple point statistics can be calculated on exploration data, such as the runs distribution and the multiple point histogram and that comparing these statistics to the suite of available TIs helps to narrow the choice of TI and provide objective support for TI selection.

### Comparing realizations with MPS

The same methodology that is used to compare exploration data to TIs can be used to compare geostatistical realizations to their parent TIs for model validation. The multiple point statistics are compared as per the methodology given above. This will be demonstrated by comparing realizations generated using SNESIM (Strebelle, 2002) to the TI that was used to generate the realizations. For comparison, realizations were also generated using sequential indicator simulation (SIS) with the variogram taken after Chevron TI #3. This TI was selected because it was ranked highly by both the multiple point histogram and the runs distribution.

All realizations generated are 78x59x50 with 75m 75m 6m block sizes respectively, and are conditioned to the 32 wells provided by Chevron. This only covers the lower half of the data set (to reduce the time required to generate the realizations). TI#3 (see Figure 15) will be used for both SNESIM and SIS to attempt to reproduce the multiple point features of the TI.

#### Generating realizations: SNSIM

Strebelle's (2002) SNESIM algorithm for multiple point stimulation will be used to generate realizations that attempt to honor the multiple point statistics of the TI shown in Figure 15. This algorithm uses the single normal equations, rather than an iterative approach, to create geostatistical realizations. The reader is referred to Strebelle (2002) for a more detailed description of the methodology.

30 realizations were generated using a 50 point multiple point template. The template is a three dimensional irregular template determined using the methodology presented by Lyster and Deutsch (2006). The parameters used in SNESIM are: maximum of 16 conditioning data; minimum of 10 data events; 4 search trees; and a search large enough to find 16 conditioning data at all locations. Figure 16 shows a typical realization.

#### Generating Realizations: SIS

The goal is to use SIS to generate realizations that contain the same features as TI#3. Therefore the variogram calculated from the TI will be used to create 30 SIS realizations. The variogram:

$$\gamma(\mathbf{h}) = 0 + 0.1sph_{\substack{ah1=15 \\ ah2=14}}(\mathbf{h}) + 0.0626sph_{\substack{ah1=50 \\ ah2=45}}(\mathbf{h}) + 0.075sph_{\substack{ah1ajor=300 \\ ah2=60}}(\mathbf{h}) \quad [9]$$

Where *ah1* is in the 90° direction, *ah2* in the 0° direction and *av* is the vertical direction. One SIS realization is shown in Figure 17.

This was generated by modeling the variogram for Chevron TI #3. Figure 18 shows that the realizations generated using SIS do reproduce the variogram for the TI, with a slightly lower correlation in the horizontal directions. However, it is clear that the structure seen in the TI, Figure 15, is missing. This is most noticeable in the horizontal slices, see Figure 17.

#### Comparing geostatistical realizations to TIs

The 30 realizations generated using SIS and the 30 generated using SNESIM (Strebelle, 2002) will be compared to the TI using the distribution of runs and the multiple point histogram. The

same comparison methodology described above will be used to compare the multiple point statistics.

The distribution of runs is considered to be a one dimensional statistic. Because most of the nonlinear features are apparent in the XY plane, see Figure 15, the runs difference was calculated for the X and Y directions and averaged to get the difference shown in Figure 19.

The multiple point histogram was also used to compare realizations. A 3x3x2 template was used: note that this is  $2^{(3 \times 3 \times 2)} = 262,144$  bins and increasing to a 3x3x3 template results in 134,217,728 bins. Since there are only 212,268 possible 3x3x2 statistics in the realizations it is unrealistic to consider a template greater than 3x3x2.

The differentiation between the simulation techniques is obvious with the multiple point histogram. SNESIM generates realizations that are more similar to the original TI, in that multiple point statistics are honored. Comparing the ranking of the two measures, Figure 21 shows that they differ significantly. The measures are most noticeably different when ranking the top ten realizations, arguably the most important realizations.

### **Discussion: Using MPS to rank realizations**

The selection of a statistic for ranking is extremely important. It is apparent that the multiple point histogram can better differentiate between the different simulation methods. The visual inspection of the SNESIM realizations indicated they are more similar to the TI, which supports the use of the multiple point histogram when ranking realizations. In this situation the runs distribution does not capture the difference between the SNESIM and SIS realizations. The multiple point histogram for a realization is well informed, as there are many repetitions of the template, whereas the role of the runs distribution is to be used when there is sparse data and a well informed multiple point histogram is not attainable (e.g. statistics calculated from drill holes). Therefore, the multiple point histogram is considered superior when ranking realizations relative to an exhaustive training image.

In this example the ranking measures were used to show that the variogram cannot capture the nonlinear features of a TI, as shown by the higher ranking of the SNESIM realizations. It is expected that this would not be the main use for ranking realizations. In practice it is unrealistic that SIS realizations would be created. It is more likely that 30 SNESIM realizations would be created and this ranking technique could be used to select the realizations that most resemble the original TI and to check for large differences between the multiple point statistics of TI and simulated realizations. If multiple point simulation is used, it is assumed that the goal is to model nonlinear and ordering features that cannot be captured otherwise, therefore this ranking measure can be used to determine which realizations have best reproduced these features borrowed from of the TI (see Figure 22)

Figure 22 shows that there is considerable difference between SNESIM realizations. The top 9 realizations have a substantially greater similarity to the original TI, see Figure 23. This could be used to determine which realizations will be kept for future analysis.

This measure should not be used to determine which models represent the p10, p50 or p90 models, but rather which realizations should be discarded as not capturing the features inherent in the TI. As shown in Figure 23 there is a significant visual difference between the realization ranked 30 and the highest ranked realization.

## Conclusions

There are a number of practical considerations that must be addressed when using multiple point statistics to compare two sets of data, whether for the comparison of exploration data with potential TIs or the validation of realizations against a TI. The first consideration is the scale of the features present: if the features are identical, but of a different size, the resulting comparison will show that the two sets of data are different. The second major consideration is which multiple point statistic to use. The multiple point histogram and the distribution of runs have been demonstrated, however, there are other multiple point statistics that could be used to compare data sets. The choice of statistic will directly impact which TI is selected or which realization is ranked highly; therefore, the statistic should be related to features that will impact the transfer function. A comparison between multiple point statistics is not sufficient evidence to select a TI, other sources of information must be used in conjunction with a statistical comparison, such as NTG, anisotropy, single point histogram and geological knowledge.

It has been shown that by using a multiple point statistical comparison, the correct geological setting was selected for the well data provided by Chevron. Moreover, the TIs that were selected by Chevron as being representative were also ranked highly by the statistical comparison. This is strong evidence that there is important information gained by performing a simple calculation of the difference between two different sets of data.

Ranking realizations is also a useful tool for determining which realizations contain the desired features present in the TI. In this context, it is recommended that the multiple point histogram be used as the ranking measure, since exhaustive TIs and realizations provide sufficient datasets for reliable calculation of the multiple point histogram.

## Future work

With the software developed for this paper it is possible to compare the distribution of runs or the multiple point histogram. No other flexibility was added. It would be useful to have a program that can compare the single point histogram, variogram, NTG, transition probabilities, runs distribution, and the multiple point histogram. Extending on this further, it would be possible to combine these multiple ranking methods by weighing each difference:

$$\text{Difference score} = 0.2(\text{variogram}) + 0.2(\text{histogram}) + 0.3(\text{runs}) + 0.3(\text{multiple point histogram})$$

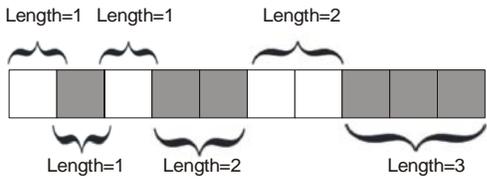
This would generate a difference score that would incorporate multiple sources of information. Weighing could be assigned based on importance. For example, if the features are linear less weigh would be placed on the multiple point statistics, if there is sparse data and the multiple point histogram is not well informed its weighting would be reduced. In this way all available objective knowledge of the exploration data can be combined into one ranking measure.

Another area of work that was not explored was developing new/different multiple point statistics that could be used for comparison. The connectivity function has been mentioned, but new measures could be developed specifically for comparing petrophysical data.

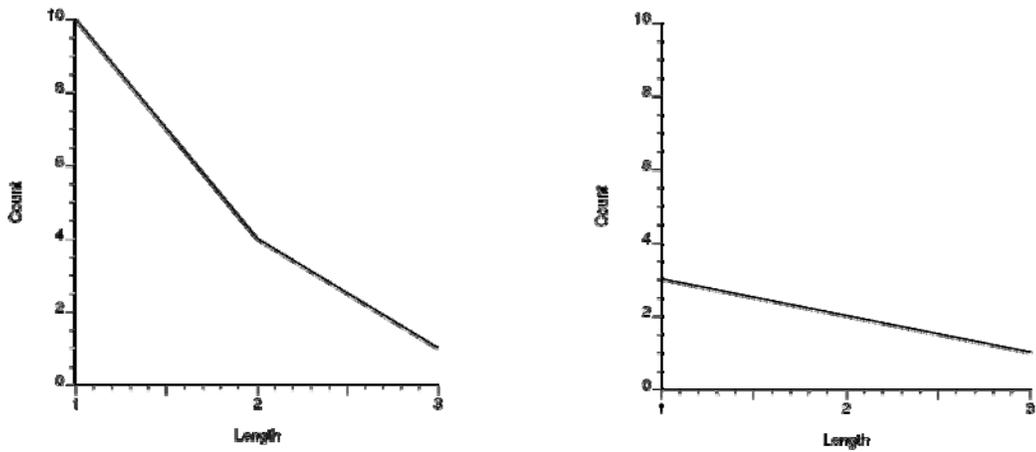
## References

Deutsch, C. V., 1992. Annealing Techniques Applied to Reservoir Modeling and the Integration of Geological and Engineering (Well Test) Data. PhD thesis, Stanford University, Stanford, CA.

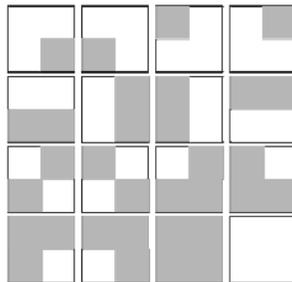
- Guardiano, F. and Srivastava, M., 1993. Multivariate geostatistics, Beyond bivariate moments, *in* Soares, A. ed., Geostatistics Troia '92: v. 1, p. 133-144.
- Journel, A. G., and Huijbregts, Ch. J., 1978. Mining Geostatistics. New Jersey, The Blackburn Press. 600 p.
- Lyster, S. and Deutsch, C. V., 2006. "An Entropy-based Approach to Establish MPS Templates", *in* Leuangthong, O. and Deutsch, C., eds., Report Seven Center for Computational Geostatistics.
- Ortiz, J. M., and Deutsch, C. V., 2003. "Indicator Simulation Accounting for Multiple Point Statistics, *in* Report Five: Center for Computational Geostatistics. 18 p.
- Pyrzcz, M.J and Strebelle, S., 2006, "Event-based Geostatistical Modeling of Deepwater Systems", GCSSEPM 2006.
- Pyrzcz, M.J., Catuneanu, O. and Deutsch, C.V., 2005, *Stochastic Surface-based Modeling of Turbidite Lobes*, American Association of Petroleum Geologists, Vol. 89., No. 2, pp 177-191.
- Pyrzcz, M. J. and Deutsch, C. V., 2003. A Library of Training Images for Fluvial and Deepwater Reservoirs and Associated Code: Center for Computational Geostatistics.
- Strebelle, S., 2002. Conditional Simulation of Complex Geological Structures Using Multiple-Point Statistics. *Math Geology*, 34 (1), pp. 1-22.



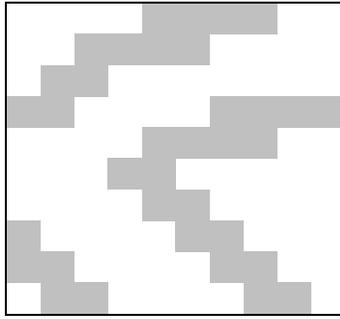
**Figure 1:** Example of calculating the distribution of runs for a well.



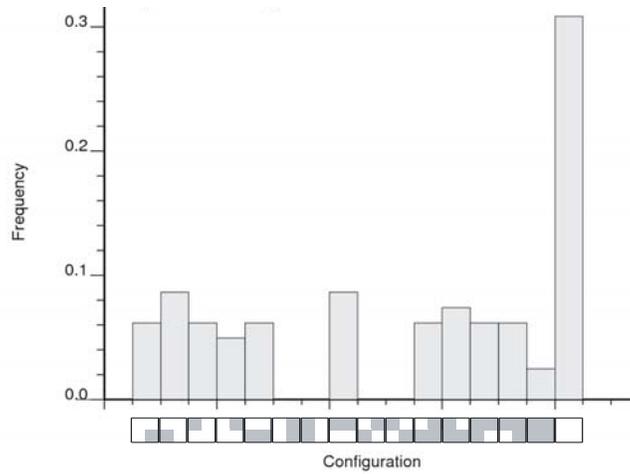
**Figure 2:** Left - cumulative distribution of runs. Right - distribution of runs.



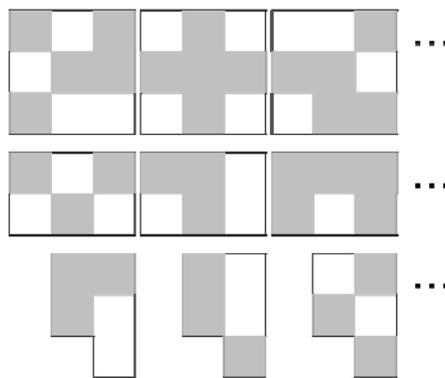
**Figure 3:** All possible arrangements of a 4 point configuration with two categories.



**Figure 4:** Example image to calculate the multiple point histogram. Figure scale: unit length, 10 by 10 blocks.



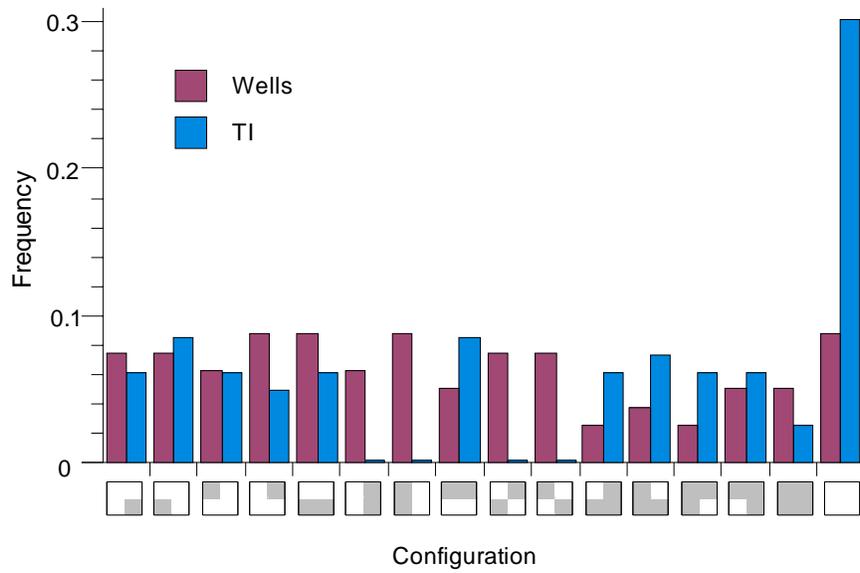
**Figure 5:** Multiple point histogram of the example image.



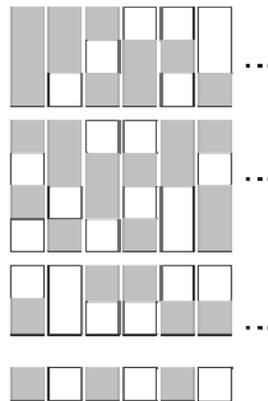
**Figure 6:** Three possible multiple point templates. Not all possible configurations of the categories are shown for each template. Above – a nine point square template. Middle – a six point rectangular template. Below – a five point irregular template.



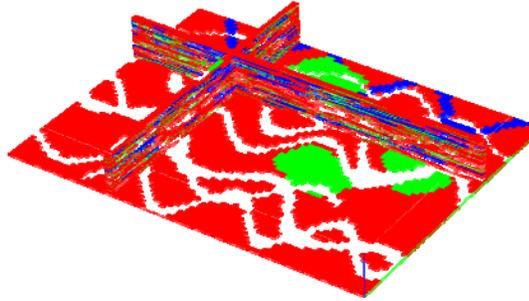
**Figure 7:** The difference between two distribution of runs. The red could be the distribution from exploration wells being compared to the green distribution from a TI.



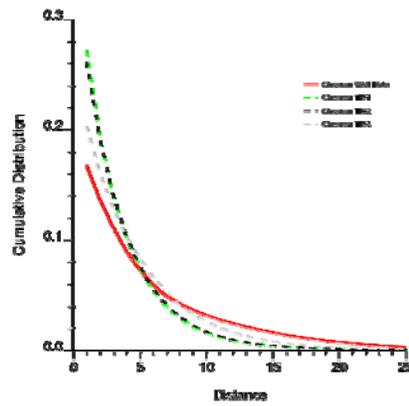
**Figure 8:** Comparing two multiple point histograms. Two distributions are shown for comparison.



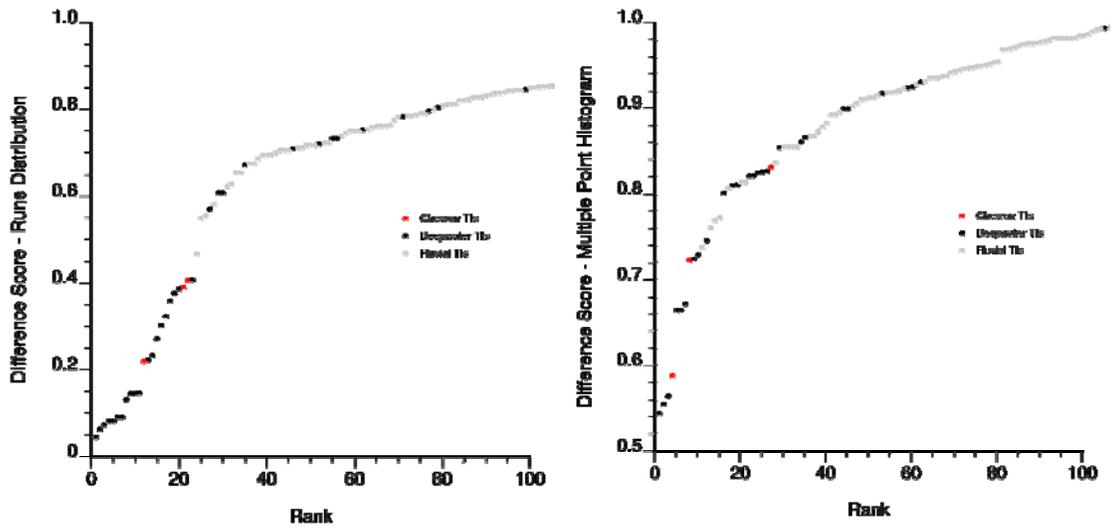
**Figure 9:** Possible one dimensional multiple point templates. Above – a continuous three point vertical template. Middle – a continuous four point vertical template. Below – an irregular three point vertical template.



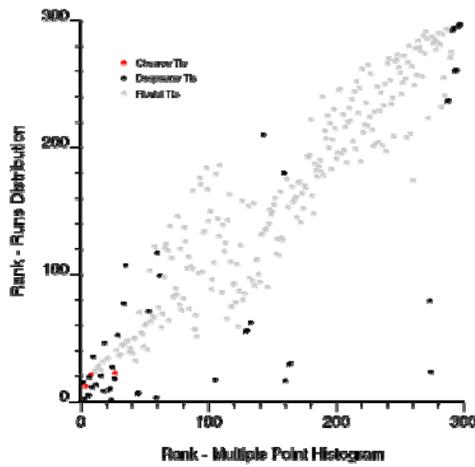
**Figure 10:** Three TIs received from Chevron, hereafter referred to as TI1 (top left), TI2 (top right), and TI3 (bottom). The facies are reduced to net/non-net categories for comparison. The red facies is considered non-net while the other facies are combined to form the net category. NTS.



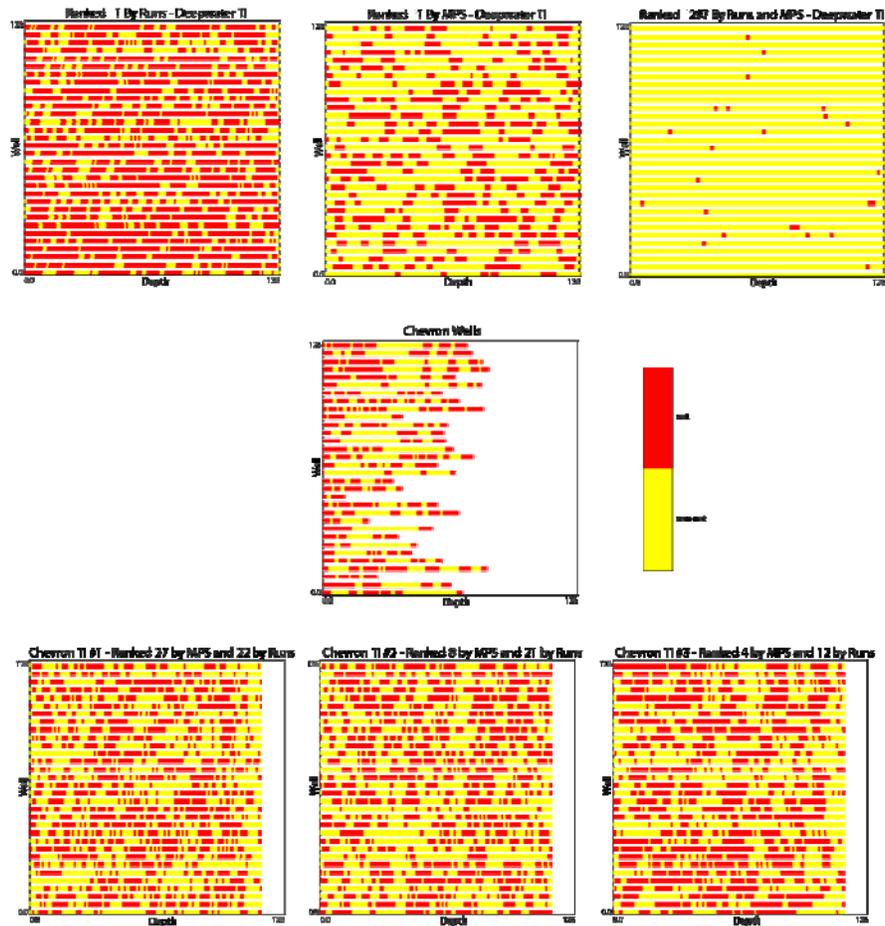
**Figure 11:** Runs distribution for the Chevron TIs.



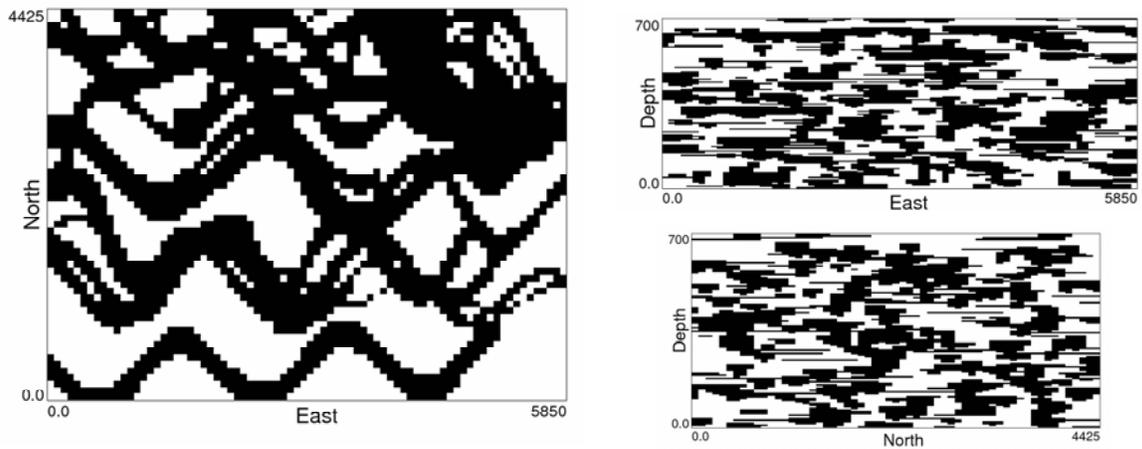
**Figure 12:** Comparing the TIs to the Chevron well data. Both the multiple point histogram and the distribution of runs is used to rank the 297 TIs. Only the 100 highest ranked TIs are shown.



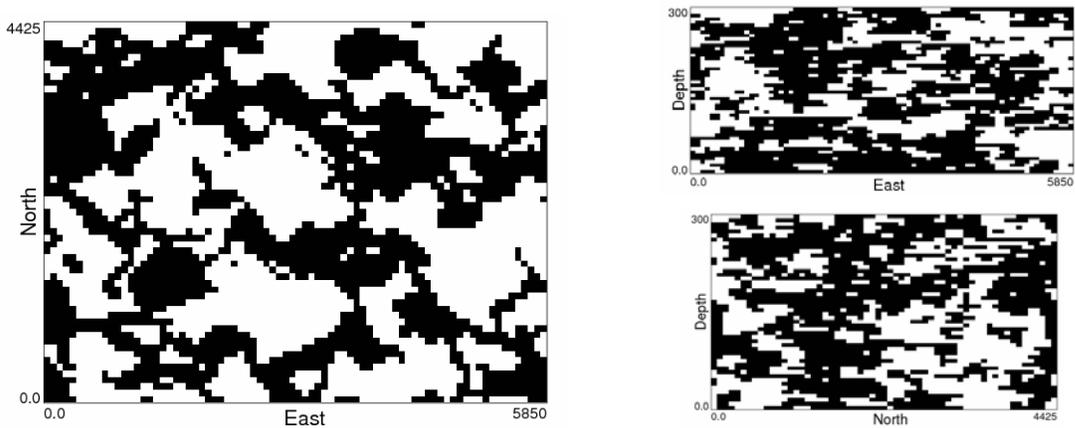
**Figure 13:** Cross plot of the rank received by each TI. Correlation = 0.90.



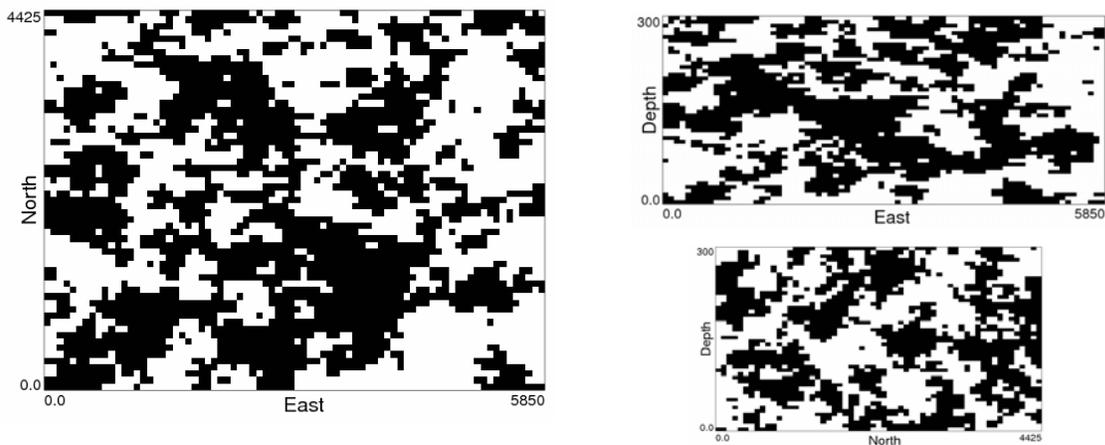
**Figure 14:** 32 wells from the highest and lowest ranked TIs, taken as columns of the TI, and the 32 wells from the data set. Both runs and MPS ranked the same TI 297<sup>th</sup>. The three Chevron TIs are also included.



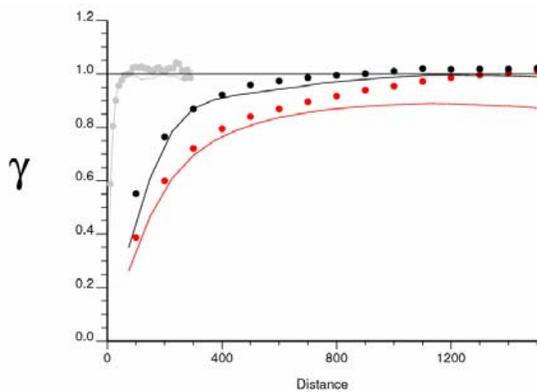
**Figure 15:** Cross sections of Chevron TI#3. The vertical is exaggerated. Black is net.



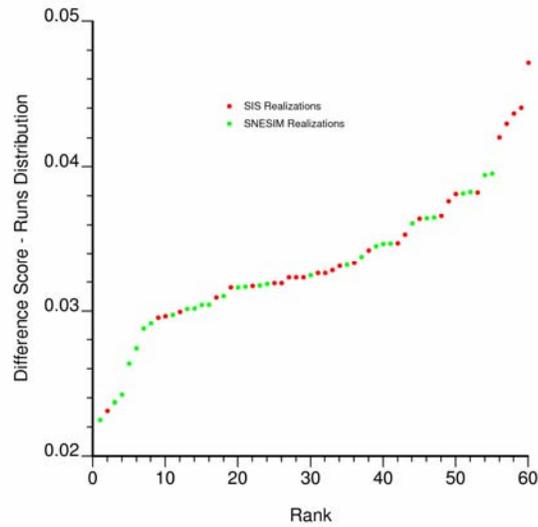
**Figure 16:** Typical SNESIM (Strebelle, 2002) realization. The vertical is exaggerated. Black is net.



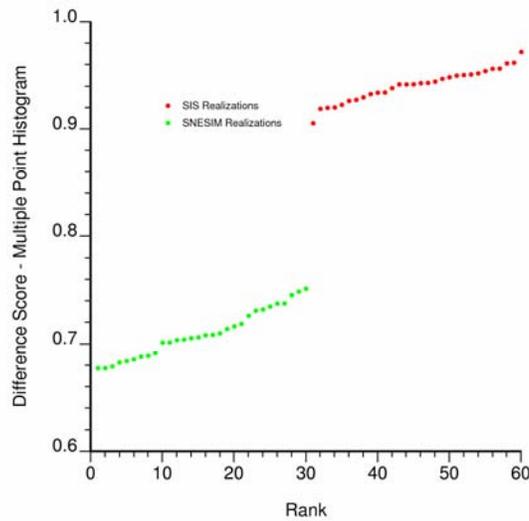
**Figure 17:** Typical SIS realization. The vertical is exaggerated. Black is net.



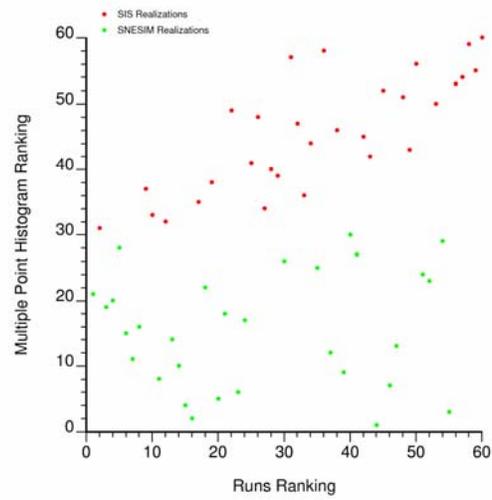
**Figure 18:** Comparing the variogram from the TI and the SIS realizations. Red represents the major direction of continuity (90°) black is the minor direction and gray is the vertical direction. Lines represent the variogram for the TI and the points are the variogram of one SIS realization.



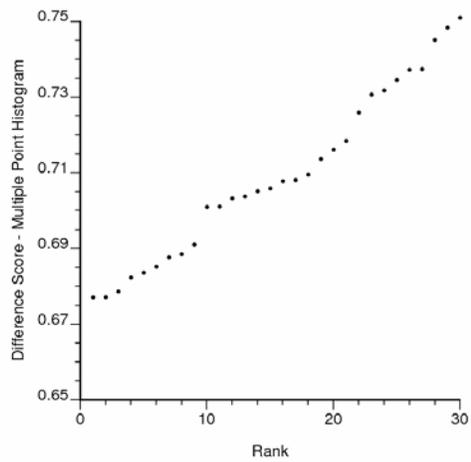
**Figure 19:** Ranking realizations based on the runs distribution.



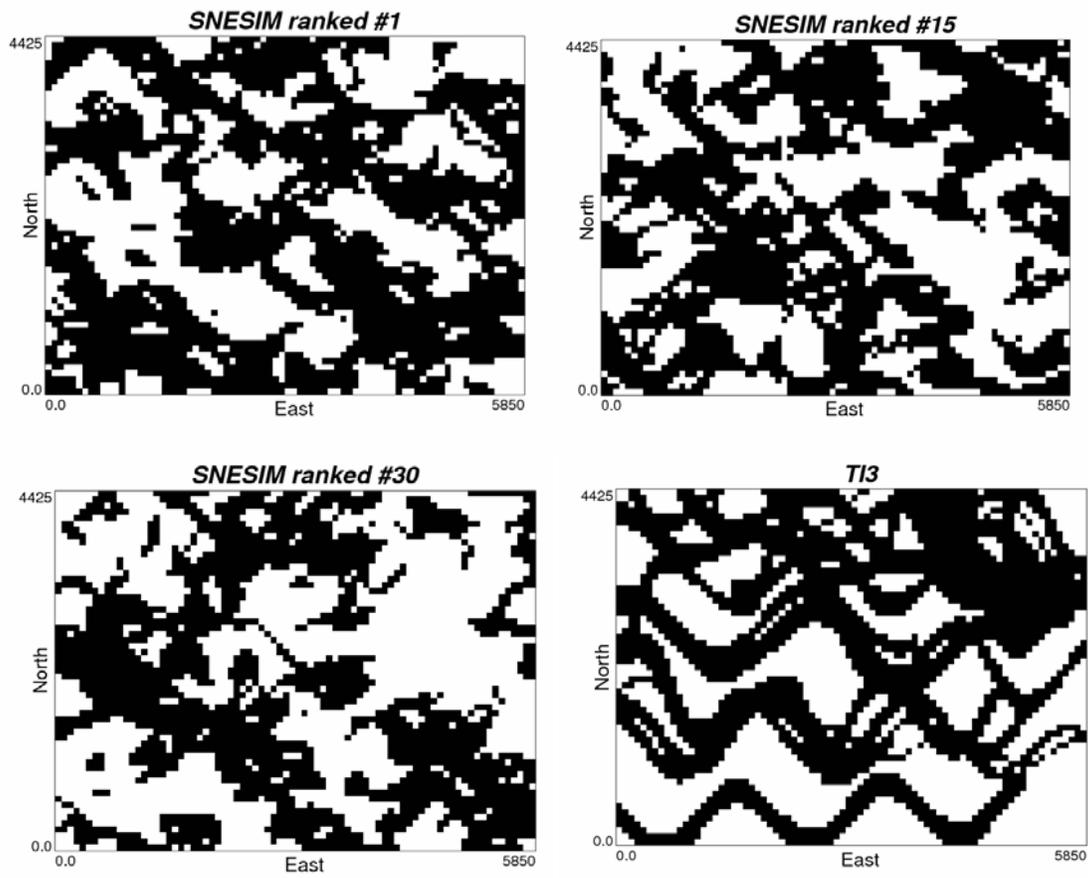
**Figure 20:** Ranking realizations based on the multiple point histogram.



**Figure 21:** Comparing the two ranking measures. Correlation = 0.43.



**Figure 22:** Ranking the 30 SNESIM realizations using the multiple point histogram.



**Figure 23:** Cross sections of ranked SNESIM realizations.